QUASILINEAR SYSTEM OF EQUATIONS FOR PARAMETERS OF DEVELOPED TURBULENCE. CALCULATION OF IRROTATIONAL FLOW DISTORTION BEHIND A GRID

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The method proposed in [1] for closing the equations of develoed anisotropic turbulence – based on hypotheses regarding the character of the dependence of the spectral tensors on the wave vector (scaling and factorization) – is used here to calculate irrotational distortions of grid-generated turbulence. The closed system of equations which is obtained is divided into two subsystems. One of the latter (for the relative intensities) is linear, while the second subsystem reduces to an independent equation for the special function A characterizing the topological structure of the flow. Calculated results are compared with examples and the theory of rapid distortion. It is shown that the result of the distortion and, in particular, the character of the asymptotes, depend to a significant extent on the turbulence structure in the incoming flow.

1. Equations in the Orientational Moments of the Spectral Tensor. A closed system of equations was obtained in [1] for secular fields of developed anisotropic turbulence. As was shown, this includes the integral scale r_c, the tensor $f_{ij}^{(0)}$ obtained by integration of the spectral function f_{ij} over all possible orientations of the wave vector k, and the function A, given by the relation

$$U_{lm}f_{ij}^{(lm)} = Af_{ij}^{(0)}. \tag{1.1}$$

The tensor $f_{ij}^{(0)}$ and the similarly determined tensor of the second-order orientational moments $f_{ij}^{(\ell m)}$ are directly connected with the Reynolds stress tensor $\langle u_iu_j\rangle$ and the "rapid" part of the pressure–strain-rate correlations $\Phi_{ij,2}$:

$$\langle u_iu_j\rangle = \alpha r_c^{-3}f_{ij}^{(0)},\quad \Phi_{ij,2} = \alpha r_c^{-3}P_{ij}, \tag{1.2}$$

where $P_{ij}\equiv\widehat{P} = U_{lm}f_{mj}^{(li)}$; $U_{lm}\equiv\widehat{U} = \partial U_l/\partial x_m$; all of the notation conforms to the notation used in [1].

The equations for $f_{ij}^{(0)}$ and r_c, obtained in [1] from the Cray equation with the use of a hypothesis on the character of the dependence of the spectral functions on the wave vector, has the form

$$U_k\frac{\partial f_{ij}^{(0)}}{\partial x_k} + \left(U_{il}f_{lj}^{(0)}\right)_s - 3Af_{ij}^{(0)} = 2\left(P_{ij}\right)_s; \tag{1.3}$$

$$U_k\frac{\partial \ln r_c}{\partial x_k} = A + 2(r_c/r_d)^{-1/\nu}t_d^{-1}. \tag{1.4}$$

In accordance with (1.4), the quantity A^{-1} is connected with the characteristic time scale of the change in r_c. Here $r_d\equiv(\eta^3/\langle\varepsilon\rangle)^{1/4}$, $t_d = (\eta/\langle\varepsilon\rangle)^{1/2}$ are the Kolmogorv scales; $\nu = 6/(4 + 3\mu)$; μ is the spectral index, characterizing the fluctuations of energy dissipation.

The following algebraic expression was obtained in [1] for $\langle\varepsilon\rangle$

$$\langle\varepsilon\rangle = 3t_d^{-1}(r_c/r_d)^{-1/\nu}\langle u_i^2\rangle/2, \tag{1.5}$$

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while for the tensor $f^{(\ell m)}_{ij}$ – the convolutions of which with $\hat{U}$ are present in the right sides of Eqs. (1.3-1.4) – the following equation was obtained

$$U_k\frac{\partial f_{ij}^{(pq)}}{\partial x_k}+\left(U_{il}f_{lj}^{(pq)}\right)_s+\left(U_{lq}f_{ij}^{(lp)}\right)_s-5Af_{ij}^{(pq)}=2U_{lm}\left(f_{mj}^{(lipq)}\right)_s. \tag{1.6}$$

In the calculations of the simplest case – axisymmetric contraction of a flow behind a grid – the components of the tensor P_{ij} are expressed through A and $f^{(0)}_{ij}$. The equation for A is derived from (1.6) by permutation of the indices p, i; q, j with U_{pi}, U_{qj}:

$$U_k\frac{\partial(\widehat{P}\widehat{U})}{\partial x_k}+4(\widehat{P}\widehat{U}^2)-9A(\widehat{P}\widehat{U})=2U_k\widehat{P}\frac{\partial\widehat{U}}{\partial x_k}. \tag{1.7}$$

After completing certain transformations, we obtain the following from (1.7) with the use of (1.3)

$$\frac{U}{\varkappa}\frac{d\bar{A}}{dx}=(\bar{A}-1)(2\bar{A}+1), \tag{1.8}$$

where $\varkappa \equiv \partial U/\partial x$; the x axis is directed along the flow; U is the corresponding component of velocity; the superimposed bar denotes that the given quantity has been changed to dimensionless form by means of $\varkappa$.

System (1.3-1.4), (1.8) is easily integrated, the integration allowing us to obtain explicit expressions for A, r_c, $f^{(0)}_{11}$, and $f^{(0)}_{ii}$ (and, thus, for the quantites $\langle u_1^2\rangle$, $\langle u_i^2\rangle$, $\langle\varepsilon\rangle$) as functions of x.

2. General Form of Irrotational Distortion. In the present study, we use the method described in [1] to calculate the flows which develop behind a grid with irrotational distortion of a general form. In this case,

$$\widehat{U}=\varkappa\begin{pmatrix}1 & 0 & 0\\ 0 & F & 0\\ 0 & 0 & -F-1\end{pmatrix},\quad -0{,}5\leqslant F\leqslant 1. \tag{2.1}$$

The bibliograph of the corresponding empirical studies is extensive; the cases most frequently investigated are those concerning axisymmetric contraction (F = –1/2) [2-6] and plane deformation, when F = 0 [7-11].

As in the calculation of axisymmetric contraction, we will limit our calculation to the turbulence characteristics on the channel axis. In this case, the above partial differential equations reduce to ordinary differential equations. Moreover, in light of the fact that most of the experimental results [7-10] are for the case when $\varkappa$ and F are constant, we will take them to be constant in the present study as well.

Flows of the given type are standard to a certain degree. Thus, on the one hand, they are most convenient for studying the interaction of pulsations with the average shear: the field of mean velocity is determined in this instance only by the geometry of the channel walls, the tensors $\langle u_iu_j\rangle$ and P_{ij} are diagonal, and the diffusion terms in the equations for $\langle u_iu_j\rangle$ are small by virtue of the condition $\sqrt{\langle u_i^2\rangle} \ll U$; on the other hand, the results being obtained here are frequently used in selecting the values of constants in various semi-empirical models [12].

Despite their relative simplicity, however, the given flows are somewhat difficult to calculate [13]. The difficulty persist despite the fact that the relative intensity of fluctuations of the different components is qualitatively described even by rapid distortion theory [14].

Let us proceed directly to the analysis of the system presented in part 1 for the case being discussed. Here, the unknowns are the intensities of all three components. Accordingly, both independent components of the tensor P_{ij} are present in Eqs. (1.3) for $f^{(0)}_{ij}$. In contrast to the case of axisymmetric contraction, single algebraic relation (1.1) is no longer adequate to determine the latter. However, it is not hard to use Eq. (1.6) to obtain an additional relation which closes the system of equations. In fact, rearranging the indices P and Q with U_{pq} in (L.6) and using (1.3), we have

$$U^2_{lm} f^{(lm)}_{ij} = B f^{(0)}_{ij}, \tag{2.2}$$

where $B \equiv A^2 + \varkappa\, dA/dt$, $i = -2 \int \varkappa dx/U(x)$ is a parameter characterizing the degree of distortion.

Using (1.8) and (2.1), it is relatively easy to show that at $F = -1/2$ Eq. (2.2) coincides with (1.1).

After performing certain calculations included in the Appendix, we use (1.1) and (2.2) to find expressions linking P_{ij} with $\bar{A}$ and $f^{(0)}_{ij}$:

$$2\bar{P}_{11} = -\frac{3}{(F-1)(F+2)}\left(\bar{B} + \bar{A} - F(F+1)\right) f^{(0)}_{11} - \frac{1}{(F-1)}\left(\bar{B} + F\bar{A} - \right.$$
$$\left. -(F+1)\right) f^{(0)}_{22} + \frac{1}{(F+2)}\left(\bar{B} - \bar{A}(F+1) + F\right) f^{(0)}_{33}; \tag{2.3}$$

$$2\bar{P}_{22} = \frac{1}{(F-1)}\left(\bar{B} + \bar{A} - F(F+1)\right) f^{(0)}_{11} + \frac{3F}{(F-1)(2F+1)}\left(\bar{B} + F\bar{A} - \right.$$
$$\left. -(F+1)\right) f^{(0)}_{22} + \frac{1}{(2F+1)}\left(\bar{B} - \bar{A}(F+1) + F\right) f^{(0)}_{33}. \tag{2.4}$$

After completing some other, rather lengthy calculations, we can use Eqs. (2.3-2.4) to reduce Eq. (1.7) to an independent equation for the function $\bar{A}$:

$$\frac{d^2\bar{A}}{dt^2} + 3\bar{A}\frac{d\bar{A}}{dt} + \bar{A}^3 - \bar{A}(F^2 + F + 1) + F(F+1) = 0.$$

Its general solution has the form ($\ell = \exp(-t/2)$)

$$\bar{A} = \frac{l^{-1/2} + FD_2 l^{-F/2} - (F+1) D_3 l^{(F+1)/2}}{l^{-1/2} + D_2 l^{-F/2} + D_3 l^{(F+1)/2}}, \tag{2.5}$$

or, equivalently, $\bar{A} = -2\ell\, d \ln u/d\ell$, where $u = \ell^{-1/2} + D_2 \ell^{-F/2} + D_3 \ell^{(F+1)/2}$; D_2 and D_3 are constants.

Using explicit expression (2.5) for $\bar{A}$, we can write Eqs. (2.3-2.4) for $\bar{P}_{ij}$ in the more compact form:

$$2\bar{\mathbf{P}} = \frac{1}{u}\widehat{M}\mathbf{f}^{(0)}. \tag{2.6}$$

Here, $\mathbf{P}$ and $\mathbf{f}^{(0)}$ are column vectors comprised of the diagonal components of the tensors P_{ij} $f^{(0)}_{ij}$; $\hat{M}$ is a matrix of the form

$$\widehat{M} = \begin{pmatrix} 3l^{-1/2} & -2(F+1) D_2 l^{-F/2} & (2F+1) D_3 l^{(F+1)/2} \\ -(F+2) l^{-1/2} & 3FD_2 l^{-F/2} & (F+2) D_3 l^{(F+1)/2} \\ (F-1) l^{-1/2} & (1-F) D_2 l^{-F/2} & -3(F+1) D_3 l^{(F+1)/2} \end{pmatrix}. \tag{2.7}$$

Finally, with allowance for (2.5-2.7), tensor equation (1.3) is easily reduced to a system of three linear homogeneous differential equations with variable coefficients:

$$4ul\frac{df^{(0)}_{11}}{dt} = \left(7l^{-1/2} + (3F-2) D_2 l^{-F/2} - (3F+5) D_3 l^{(F+1)/2}\right) f^{(0)}_{11} -$$
$$- 2(2F+1) D_2 l^{-F/2} f^{(0)}_{22} + 2(2F+1) D_3 l^{(F+1)/2} f^{(0)}_{33};$$
$$4ul\frac{df^{(0)}_{22}}{dt} = -2(F+2) l^{-1/2} f^{(0)}_{11} + \left((3-2F) l^{-1/2} + 7FD_2 l^{-F/2} - \right.$$

$$-(5F+3)D_3l^{(F+1)/2})f_{22}^{(0)}+2(F+2)D_3l^{(F+1)/2}f_{33}^{(0)},$$

$$4ul\frac{df_{33}^{(0)}}{dt}=2(F-1)l^{-1/2}f_{11}^{(0)}-2(F-1)D_2l^{-F/2}f_{22}^{(0)}+$$

$$+((2F+5)l^{-1/2}+(5F+2)D_2l^{-F/2}-7(F+1)D_3l^{(F+1)/2})f_{33}^{(0)}. \qquad (2.8)$$

In the special case ($F = -0.5$), Eq. (2.5) and the first equation of (2.8) reduce to relations found in [1]:

$$\overline{A}=\frac{1}{2}\left(\frac{3}{1+\beta l^3}-1\right),\quad U\frac{df_{11}^{(0)}}{dx}=\overline{A}f_{11}^{(0)}\quad (\beta=D_2+D_3).$$

3. Problem of Initial Conditions. The equations obtained in part 2 actually have no empirical constants, while the initial values of the functions $f_{ij}{}^{(0)}$ are easily found from the experimental conditions on the basis of the first equation of (1.2). At the same time, difficulties are encountered when attempting to find the initial values of the function A and its derivative (or, equivalently, the parameters D_2 and D_3).

First of all, it is necessary to set up a fairly sophisticated spectral experiment in order to determine these values. In connection with this, we will obtain an explicit representation for the constants D_2 and D_3 so as to clarify their importance in relation to the turbulence spectrum. With allowance for Eqs. (2.5), relations (A.1), (A.2) presented in the appendix for the components of the tensor $F\binom{0}{ij}$ can be written in the form

$$f_{ij}^{(11)}=\frac{l^{-1/2}}{u}f_{ij}^{(0)},\quad f_{ij}^{(22)}=\frac{D_2l^{-F/2}}{u}f_{ij}^{(0)}. \qquad (3.1)$$

When $\ell = 1$, we find from (3.1) that

$$\frac{1}{1+D_2+D_3}=\frac{f_{ij}^{(11)}(1)}{f_{ij}^{(0)}(1)},\quad \frac{D_2}{1+D_2+D_3}=\frac{f_{ij}^{(22)}(1)}{f_{ij}^{(0)}(1)}. \qquad (3.2)$$

In accordance with (3.2), the constants D_2 and D_3 are determined by the initial values of the components of the tensor of second-order orientational moments.

A second difficulty is connected with the specifics of the given class of slows, specifically: the condition $\varkappa$ = const implies a jump in the derivative of mean velocity at the beginning of the distortion section. As a result, the derivatives of $\langle u_iu_j\rangle$ and r_c will also probably be discontinuous at $t = 0$. Proceeding on the basis of Eq. (1.4), we can reach a similar conclusion in regard to the function A. This means that D_2 and D_3 cannot be unambiguously determined from the characteristics of the incoming flow in the present case.

However, there are other general considerations which make it possible to specify the region of possible values of D_2 and D_3. As a starting point, we will use the well-known inequality [15]

$$F_{ij}\zeta_i\zeta_j^*\geqslant 0 \qquad (3.3)$$

(ζ is an arbitrary vector). One consequence of Eq. (3.3) is the relation $\langle u_1^2\rangle\langle u_2^2\rangle \geq \langle u_1u_2\rangle^2$ [16], which is essentially a constraint on the values that can be taken by the zeroth-order moments. It is not difficult to obtain a similar relation for second-order moments. In particular, choosing as ζ_i the vector $K_{i\ell}\theta_\ell$ (K denotes a diagonal matrix), after we integrate the left side of inequality (3.3) over all θ we obtain the form $K_{i\ell}f\binom{\ell m}{ij}K_{jm}$. It follows from the conditions of its positive-definiteness that $f\binom{11}{11}f\binom{22}{22} \geq (f\binom{12}{12})^2$. For isotropic (in the sense $f\binom{0}{ij}_{\ell=1} \sim \delta_{ij}$) initial conditions with $\ell = 1$, with allowance for (3.2) we obtain the following expression from the last inequality

$$(D_3-D_2-1)^2\leqslant 4D_2. \qquad (3.4)$$

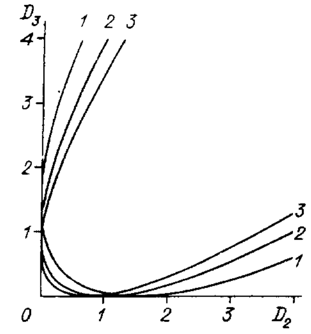

Fig. 1

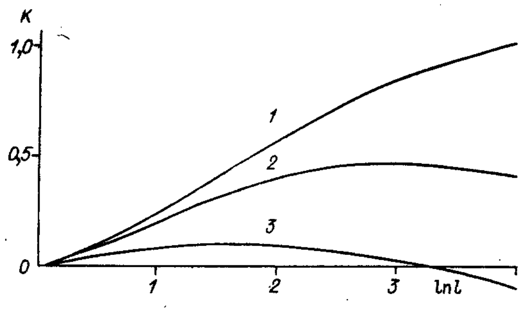

Fig. 2

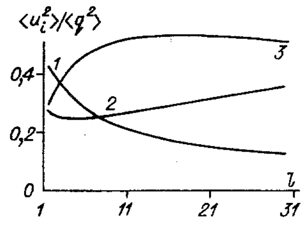

Fig. 3

The corresponding region of possible values of D_2 and D_3 is shown in Fig. 1. It is bounded by a parabola whose symmetry axis coincides with the straight line $D_2 = D_3$ (line 1). Similar results are obtained when the flow at the inlet is asymmetrical: $f^{(0)}_{22} = f^{(0)}_{33}$, $f^{(0)}_{11} = a f^{(0)}_{22}$ (i.e., $\langle u_1^2\rangle/\langle u_2^2\rangle = a$). As an example, parabolas corresponding to the values $a = 1.5$ and 0.75 (lines 2 and 3) are shown in Fig. 1.

4. Conditions of Axial Symmetry. The problem of finding the values of D_2 and D_3 has an additional aspect which is deserving of separate consideration. If we proceed as in rapid distortion theory and use isotropic parameterization $F_{ij} \sim (\delta_{ij} - \theta_i\theta_j)$ at $t = 0$ for the spectral tensor, then moments of any order can be calculated directly. However, it was shown in [1] that the concept of isotropic turbulence constitutes a very coarse model of actual turbulent flow, which can evidently not generally occur in the given case. This conclusion finds support from the fact that, within the framework of the proposed method, axisymmetric models also prove inadequate. Specifically, if the axis x_1 corresponds to the direction of undistorted flow, then at $x_1 = 0$ it follows from the condition of axial symmetry that

$$f_{22}^{(0)} = f_{33}^{(0)},\quad f_{ij}^{(22)} = f_{ij}^{(33)},\quad f_{j2}^{(i2)} = f_{j3}^{(i3)}. \tag{4.1}$$

It is not difficult to find from the last equation of (4.1) that $P_{22}(0) = P_{33}(0)$. From this, with allowance for (2.6) and (2.7) we obtain

$$D_2 = \frac{a(2F+1) - (4F+5)D_3}{4F-1}. \tag{4.2}$$

With allowance for Eqs. (3.2), the second condition of (4.1) yields $D_2 = D_3$. As a result, we find from (4.2) that $D_2 = D_3 = a/4$.

Thus, at first glance, assuming that the incoming flow is axially symmetric makes it possible to unambiguously determine the parameters D_2 and D_3. Moreover, with isotropic (relative to $f^{(0)}_{ij}$) initial conditions, when $a = 1$, $D_2 = D_3 = 0.25$ (this point coinciding with the vertex of the parabola bounding the region (3.4)), while $\beta = 0.5$. Here, the last value differs only slightly from that obtained in [1] from a comparison with experimental data on axisymmetric contraction.

However, this difference is of a fundamental character: near values of β close to 0.5, there is a significant change in the character of the asymptote of the quantity $\langle u_i^2\rangle/\langle u_1^2\rangle$, i.e., at $\beta = 0.5$ we find from Eq. (42) in [1] that $\langle u_i^2\rangle/\langle u_1^2\rangle = 1 + 2\ell^{-3} \underset{\ell\to\infty}{\to} 1$. At the same time, at $\beta > 0.5$, this magnitude decreases monotonically to zero. This is the pattern seen in most experiments. The solutions of system (2.8) are similarly (relative to the parameters D_2 and D_3) sensitive in the general case, when $F \neq -0.5$.

In sum, we can make the following conclusion: the character of energy redistribution among the components as a result of external distortions depends to a large extent on satisfaction of conditions of axisymmetry (more exactly, the degree to which they are not satisfied) of the incoming flow in relation to the components of the tensor $f_{ij}^{(\ell m)}$. This conclusion is indirectly supported by the appreciable scatter of the empiricial data for convergent nozzles and, in particular, the anomalous results in [4]. An increase in the components $\langle u_1^2\rangle$ was seen in the latter study on the contraction section. The above conclusion is also consistent with the conclusions reached in [17-19] on the topological nontriviality of the turbulence structure and the substantial effect of the corresponding parameters on evolution of the flow.

5. Some Results of Numerical Calculations. For the sake of definiteness, in this section we present results pertaining to the case $F = 0$. As was noted in part 4, while solving system (2.8) we found that there was a high degree of sensitivity with regard to the values of D_2 and D_3. As an example, Fig. 2 shows the dependence of the anisotropy parameter $K = (\langle u_3^2\rangle - \langle u_1^2\rangle)(\langle u_3^2\rangle + \langle u_1^2\rangle)$ on $\ln \ell$ for different D_3 when $D_2 = 1$. Curves 1-3 correspond to $D_3 = 0$, 0.1, and 0.5.

A significant role is played by the point $D_3 = 0$, $D_2 = 1$ in the region of possible values of D_2 and D_3: at such parameters, the value of K and the relative intensities of the different components change monotonically – as in rapid distortion theory. However, this is the extent to which the results agree with the given theory. For example, the asymptotic value of $\langle u_3^2\rangle/\langle q^2\rangle$ ($\langle q^2\rangle \equiv \langle u_i^2\rangle$) turns out to be equal to one, rather than 0.5.

As is known [9-11], the rate of change in intensities is lower in experiments than the rate predicted by rapid distortion theory. Moreover, the estimate 0.5-0.7 is obtained for the asymptotic value of $\langle u_3^2\rangle/\langle q^2\rangle$. Such features of the empirical data are reproduced by the calculation if values somewhat greater than zero are used for D_3.

As an illustration, we examined a third series of experiments [9]: plane distortion, anisotropic initial conditions, and the attainment of a presumably asymptotic state characterized by relative intensities of 0.1, 0.37, and 0.53 at the end of the distorting section (ℓ 7.2). Townsend [7] obtained a somewhat different estimate for these values: 0.19, 0.33, and 0.48. This difference may be evidence of the high degree of sensitivity in relation to initial anisotropy. Figure 3 shows the results of calculations corresponding to the conditions in the indicated experiments. In the figure, the number of the curve corresponds to the index i in $\langle u_i^2\rangle/\langle q^2\rangle$ (with no summation carried out over i). Also, $D_2 = 1$. It is evident that the value of $\langle u_3^2\rangle/\langle q^2\rangle$ at $\ell \geq 8$ actually changes little. However, this region corresponds to a shallow maximum rather than an asymptote. The value of $\langle u_3^2\rangle/\langle q^2\rangle$ at the point of the maximum depends to a considerable extent on D_3. It coincides with 0.53 if we choose $D_2 = 0.035$. The corresponding relative intensities $\langle u_1^2\rangle/\langle q^2\rangle$ and $\langle u_2^2\rangle/\langle q^2\rangle$ are equal to 0.15 and 0.30 in this case, these figures agreeing rather well with the above-cited empirical estimates.

On the whole, the change in intensities which occurs at $D_3 \neq 0$ takes place in a complex manner: the values of $\langle u_3^2\rangle/\langle q^2\rangle$ and $\langle u_2^2\rangle/\langle q^2\rangle$ pass through maximum and minimum points and then asymptotically approach zero and one. As was noted in [9], even calculations performed in accordance with rapid distortion theory indicated the potential for such an unexpected evolution and the existence of extreme points. However, the theory indicated this only for values of F close to one. In addition, the function $\langle u_3^2\rangle/\langle q^2\rangle$ was seen to have a maximum on several experimental curves in [7, 8]. However, it had been integrated as a result of perturbations at the outlet and, moreover, the degree of flow distortion was relatively slight ($\ell = 4$ and 6). On the whole, the question of the possibility of reversal of the asymptote and the existence of anomalous intercomponent energy transfer for high degrees of deformation remains unanswered.

Appendix. With allowance for the identity $f_{ij}^{(\ell\ell)} = f_{ij}^{(0)}$, we can easily use Eqs. (1.1) and (2.2) to obtain algebraic relations expressing the components of a second-rank orientation tensor in terms of $f_{ij}^{(0)}$, A, and B:

$$f_{ij}^{(11)} = \frac{1}{(F+2)(1-F)}\left(\overline{B} + \overline{A} - F(F+1)\right)f_{ij}^{(0)}; \tag{A.1}$$

$$f_{ij}^{(22)} = \frac{1}{(2F+1)(F-1)}\left(\overline{B} + F\overline{A} - (F+1)\right)f_{ij}^{(0)}. \tag{A.2}$$

On the other hand, inserting matrix (2.1) into the definition $P_{ij} = U_{\ell m} f_{mj}^{(\ell i)}$ with allowance for the identity $f_{j\ell}^{(ii)} = 0$ (the incompressibility condition), we have

$$\overline{P}_{ij} = (F+2)f_{1j}^{(1i)} + (2F+1)f_{2j}^{(2i)}. \tag{A.3}$$

Taking advantage of the symmetry of $f_{ij}^{(\ell m)}$ with respect to its superscripts and subscripts and making use of the identity $P_{ii} = 0$, we can use (A.3) to derive two equations which link P_{11} and P_{22}:

$$\overline{P}_{22} - \frac{F+2}{2F+1}\overline{P}_{11} = -\frac{(F+2)^2}{2F+1}f_{11}^{(11)} + (2F+1)f_{22}^{(22)},$$

$$-\frac{3}{F+2}\overline{P}_{22} - \overline{P}_{11} = \frac{(F-1)^2}{F+2}f_{22}^{(22)} - (F+2)f_{33}^{(33)}.$$

After completing some simple but lengthy calculations using (A.1) and (A.2), we can use the above relations to obtain Eqs. (2.3-2.4) from the main text.

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PHENOMENOLOGICAL DESCRIPTION OF TWO-VELOCITY MEDIA WITH RELAXING TANGENTIAL STRESSES

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Tangential stresses are generated and subsequently relax during the filtration process of high-temperature solutions (or melts) through an enclosing island. The stresses generated as well as their relaxation dynamics start determining, in turn, the filtration mechanics of the fluid phase leading to a self-contained interaction process of the continua under consideration.

The concept of effective elastic deformation was suggested in [1] to describe the relaxation of tangential stresses is a viscoelastic medium. By introducing it the authors succeeded in generalizing the Maxwell relaxation model to the case of substantial medium deformation. This is one of the principal approaches in nonlinear filtration theory. The generalization of the Maxwell model to filtration media within the approximations of small deformations and low velocities of the filtering fluid was investigated numerous times in the literature (see, for example, [2]). To the best of the authors' knowledge, the extension of the Maxwell model to the case of nonlinear island deformation and high fluid filtration rates is not available in the literature.

Under conditions of filtration of a viscous fluid through viscoelastic medium the effective elastic deformation must be introduced somewhat differently than was done in [1]. A theory using the concept of effective elastic deformation must be compatible with general physical requirements: conservation laws and the Galileo relativity principle.

Below we obtain a system of differential equations, describing the relaxation of tangential stresses of a viscoelastic island during its self-consistent interaction with a filtering viscous fluid. The necessary requirement on the initial deformation of the state of the medium is established. The system of equations describes both compact and noncompact two-velocity continua.

For a basis of the general theory one must construct a formalism of elastic interaction of the island with the filtering fluid in the reversible hydrodynamic approximation. To describe the filtration process within the continuum approach we introduce two velocity fields: $\mathbf{u}$ – the velocity of motion of an elastic continuum with particle density ρ_1, and $\mathbf{v}$ – the velocity of motion of a fluid with partial density ρ_2, filtering through the elastic continuum. Two such mutually penetrable continua can interact through a friction force $\mathbf{f}$, which is not present in the reversible approximation, and a reaction force being in hydrodynamics proportional to gradients of thermodynamic quantities. Besides, the set of two continua is a hydrodynamic system for which conservation laws are valid, being in the case of reversible motion

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